Statistics

Lecture 12

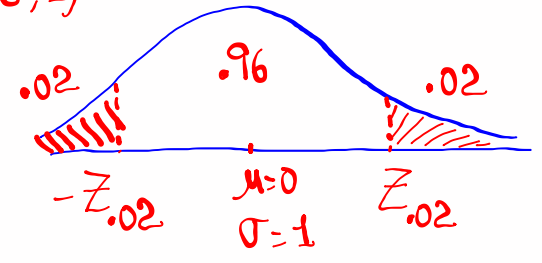
Feb 19-8:47 AM

Find $Z_{\alpha/2}$ for 96% Conf. level. $1 - .96 = .04$

Middle Area $.04 \div 2 = .02$

$Z_{\alpha/2} = Z_{.02} = \text{invNorm}(.98, 0, 1)$ .96

$= \boxed{2.054}$



Apr 30-6:50 PM

In a Survey of 275 students, 32% of them were full-time students.

$n = 275$ $\hat{p} = .32$

$x = n\hat{p} = 275(.32) = 88$

if decimal → Round-up

Find Confidence interval for the prop. of all students that are full-time students.

No C-level ⇒ use .95

1-PropZInt

$x = 88$

$n = 275$

C-level: .95

Calculate

$.265 < p < .375$

27% < p < 38%.

We are 95% Confident that between 27% and 38% of all students are full-time student.

$E = \frac{.375 - .265}{2} = .055 \approx 5.5\%$

$\hat{p} = \frac{.375 + .265}{2} = .32 = 32\%$

Apr 30-6:53 PM

How to determine minimum Sample Size for Constructing Conf. interval for Pop. Prop.:

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow n = \hat{p}\hat{q} \cdot \left(\frac{z_{\alpha/2}}{E}\right)^2$$

If $\hat{p}$ & $\hat{q}$ unknown, use .5 for each

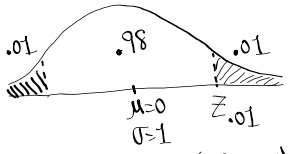
$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

If decimal ⇒ Always Round-up

Apr 30-7:03 PM

Suppose $\hat{p} = .4$ and $E = 4\%$. Find min. Sample Size needed to construct 98% C-level for Pop. Prop.

$\hat{p} = .4$, $\hat{q} = .6$, $E = .04$



$z_{.01} = \text{invNorm}(.99, 0, 1)$

$= 2.326$

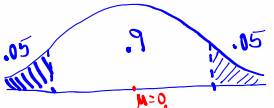
$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

$= (.4)(.6)\left(\frac{2.326}{.04}\right)^2$

$= 811.5414$

$n = 812$

Redo with 90% C-level and margin of error not to exceed 5%.



$z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

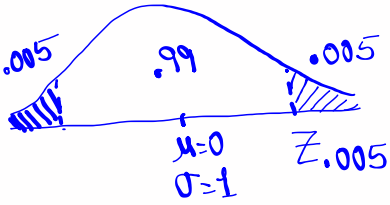
$= (.4)(.6)\left(\frac{1.645}{.05}\right)^2$

$= 259.7784$

$n = 260$

Apr 30-7:06 PM

Find minimum Sample Size needed to Construct 99% Conf. interval for pop. Prop. and error not to exceed 8%.



$z_{.005} = \text{invNorm}(.995, 0, 1)$

$= 2.576$

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

If $\hat{p}$ & $\hat{q}$ unknown, use .5 for each

$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$

Always round-up if decimal.

$n = .25\left(\frac{2.576}{.08}\right)^2 = 259.21$

$n = 260$

Apr 30-7:16 PM

Given the Conf. interval below

$75.8 < \mu < 94.2$

1) find margin of error

$E = \frac{94.2 - 75.8}{2} = \boxed{9.2}$

2) find point-Estimate

$\bar{x} = \frac{94.2 + 75.8}{2} = \boxed{85}$

Apr 30-7:24 PM

Given $n = 32$, $\bar{x} = 86$, $\sigma = 10$, C-level: .96

Find Conf. interval for pop. mean μ.

σ known $\Rightarrow$ Z Interval

σ unknown $\Rightarrow$ T Interval

$\Rightarrow$ inpt: Stats

$82.369 < \mu < 89.631$

Since point-estimate $\bar{x}$ is a whole # $\Rightarrow$ $\boxed{82 < \mu < 90}$

$E = \frac{90 - 82}{2} = \boxed{4}$

$\bar{x} = \frac{90 + 82}{2} = \boxed{86}$

Apr 30-7:27 PM

Given: $n=18$, $\bar{x}=34.6$, $S=8.5$

Find Conf. interval for pop. mean μ.

No C-level ⇒ use .95

σ Known ⇒ Z Interval

σ unknown ⇒ T Interval

inpt: Stats

$30.373 < \mu < 38.827$

Since $\bar{x}$ is one-decimal ⇒ Round to 1-decimal

$30.4 < \mu < 38.8$

$E = \frac{38.8 - 30.4}{2} = 4.2$

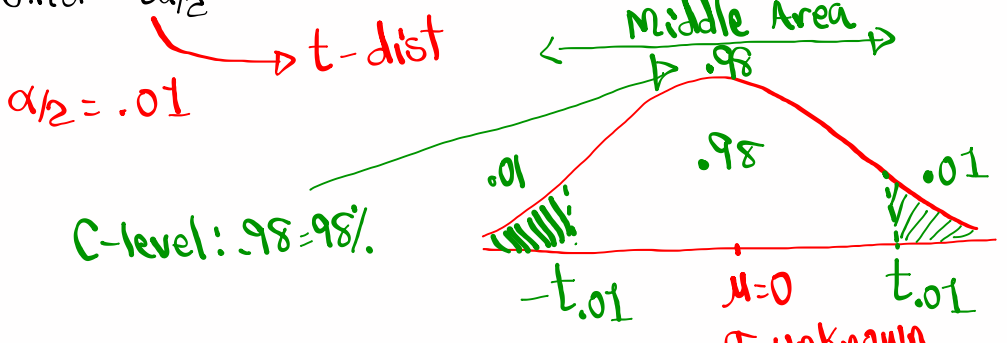
$\bar{x} = \frac{38.8 + 30.4}{2} = 34.6$

Apr 30-7:32 PM

Find $t_{\alpha/2}$ for $\alpha=.02$ and $df=11$.

t-dist

$\alpha/2 = .01$



C-level: .98 = 98%

σ unknown

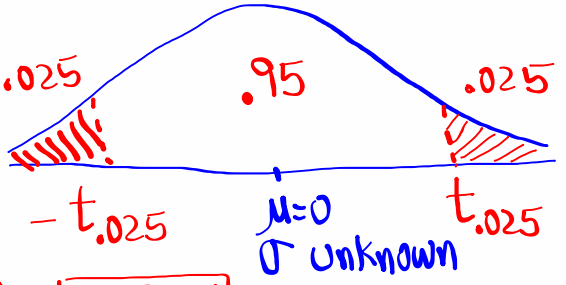
df = 11

$t_{.01} = \text{invT}(.99, 11) = 2.718$

Apr 30-7:38 PM

Find $\pm t_{\alpha/2}$ for 95% C-level with df=99.

Middle Area = .95



σ unknown

df=99

$t_{.025} = invT(.975, 99) = \boxed{1.984}$

As df increase $\Rightarrow t_{\alpha/2} \approx Z_{\alpha/2}$

Apr 30-7:42 PM

Consider the Sample below

75	86	68	98
70	100	90	78
80	55	70	80
92	75	85	90

Find

1) $\bar{x} = 80.75 \approx 81$ } Round to whole #

2) $s = 11.953 \approx 12$ } Round to whole #

3) $s^2 = \frac{2143}{15}$ } Reduced fraction

4) find 90% C.I. for one Pop. mean μ.

σ Known $\Rightarrow$ Z Interval

σ Unknown $\Rightarrow$ T Interval

Since $\bar{x}$ is a whole #, Round to whole #

$75.741 < \mu < 86.259$

$\boxed{76 < \mu < 86}$

$E = \frac{86-76}{2} = 5$

$\bar{x} = \frac{86+76}{2} = 81$

Apr 30-7:46 PM

How to determine minimum Sample Size needed when constructing Conf. interval for Pop. mean:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$n = \left(\frac{z_{\alpha/2} \cdot s}{E} \right)^2$$

If σ is unknown, use s instead.

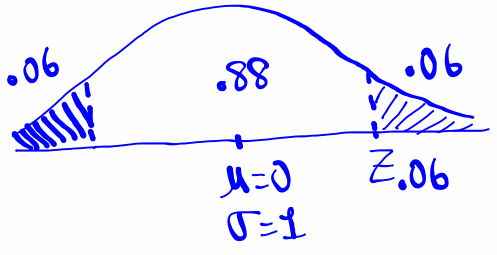
Always round up when decimal.

Apr 30-8:08 PM

Given $\sigma = 12.5$, $E = 5$ and C-level: .88

Find min. Sample Size n for constructing C.I. for Pop. mean.

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$



$$= \left(\frac{1.555 \cdot 12.5}{5} \right)^2$$

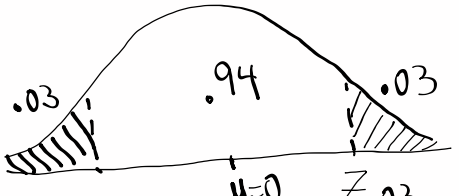
$$= 15.113$$

$z_{.06} = \text{invNorm}(.94, 0, 1) = 1.555$

$n = 16$

Apr 30-8:12 PM

Redo last problem with 94% C-level and error not to exceed 2.5.



$z_{.03} = \text{invNorm}(.97, 0, 1) = 1.881$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

$$= \left(\frac{1.881 \cdot 12.5}{2.5}\right)^2$$

$= 88.454\ldots$

$n = 89$

Apr 30-8:17 PM

Find min. Sample Size needed to construct C.I. for pop. mean with margin error not to exceed 7.5 with Sample Standard dev. of 14.8.

Find n

$S = 14.8$

σ not given, use S

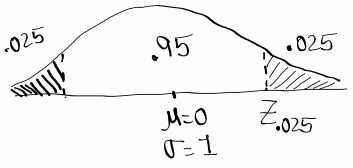
C-level not given → use .95

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

$$n = \left(\frac{1.960 \cdot 14.8}{7.5}\right)^2$$

$= 14.959\ldots$

$n = 15$



$z_{.025} = \text{invNorm}(.975, 0, 1) = 1.960$

SG 22 & SG 23 ✓

Apr 30-8:21 PM

Chi – Square dist.

χ^2

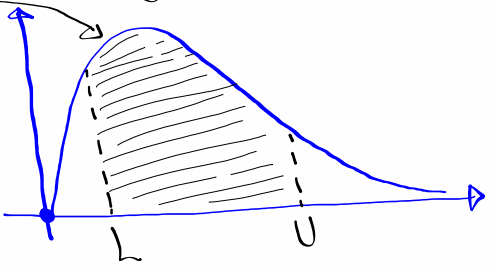
1) Graph begins at 0, and it is skewed to the right.

2) Not Symmetric, total Area = 1

3) It comes with degrees of freedom.

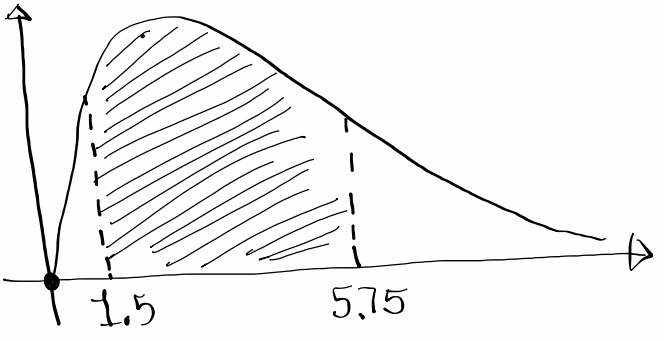
2nd VARS ↓

χ^2cdf(L, U, df)



Apr 30-8:28 PM

find $P(1.5 < \chi^2 < 5.75)$ with df = 8.



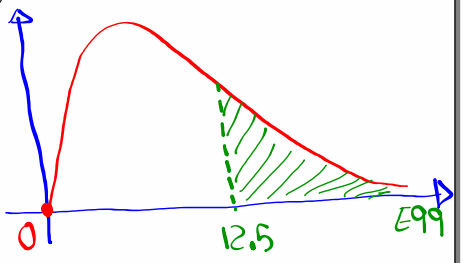
χ^2cdf(1.5, 5.75, 8) = .317 ✓

L U df

Apr 30-8:32 PM

Find $P(\chi^2 > 12.5)$ with $df = 7$.

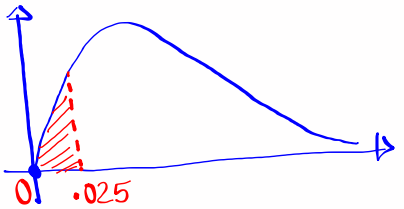
shade to the right



$= \chi^2cdf(12.5, E99, 7) =$ **.085**

Find $P(\chi^2 < .025)$ with $df = 9$.

shade left



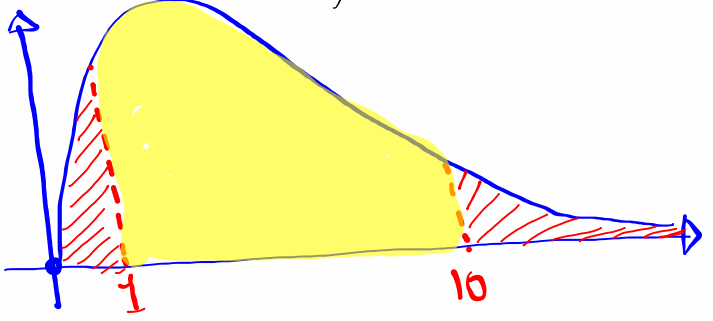
$\chi^2cdf(L, U, df)$

$\chi^2cdf(0, .025, 9) =$ 5.162×10^{-11} S.N.

Apr 30-8:35 PM

Find $P(\chi^2 < 1$ or $\chi^2 > 10)$ with $df = 6$.

Total Area



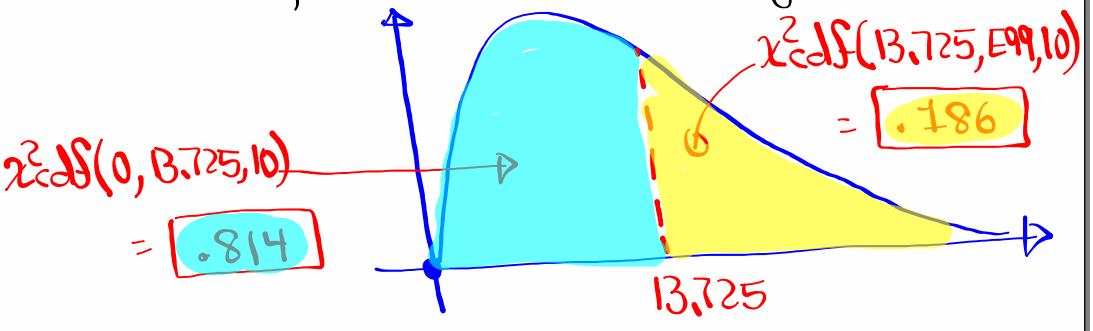
$= 1 - \chi^2cdf(1, 10, 6) =$ **.139**

Apr 30-8:41 PM

Given $\chi^2 = 13.725$, $df = 10$

Find the area on each side of $\chi^2 = 13.725$,

then multiply the smaller area by 2.



$2(.186) = .372$

Apr 30-8:44 PM

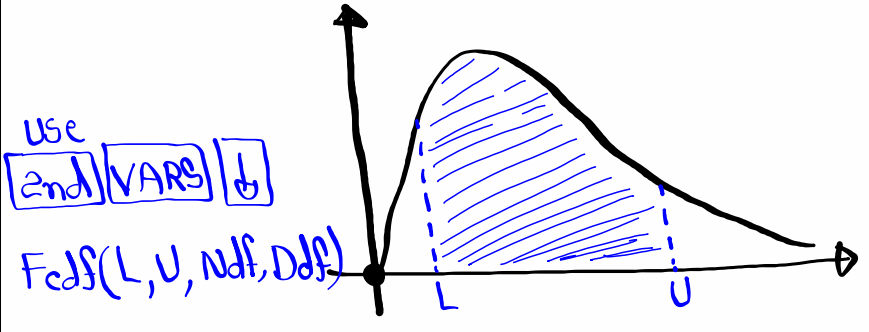
F-Dist

1) It is similar to χ^2-Dist.

2) It comes with two degrees of freedom

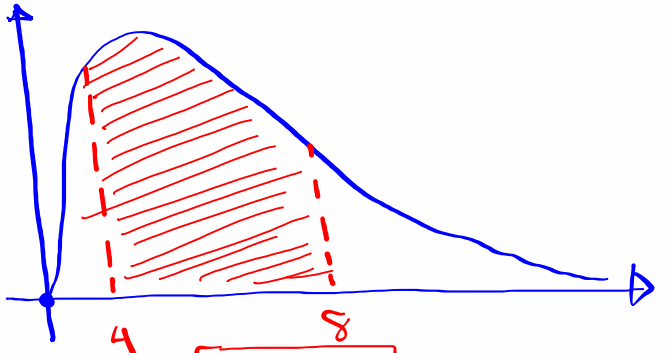
Ndf Numerator df

Ddf Denominator df



Apr 30-8:49 PM

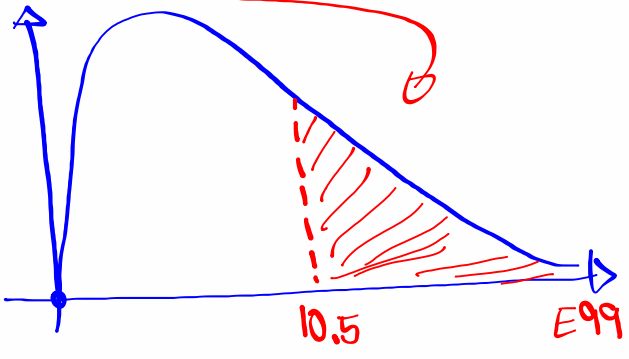
Find $P(4 < F < 8)$ with $Ndf = 3$ & $Ddf = 25$.



$fcdf(4, 8, 3, 25) = .018$

Apr 30-8:53 PM

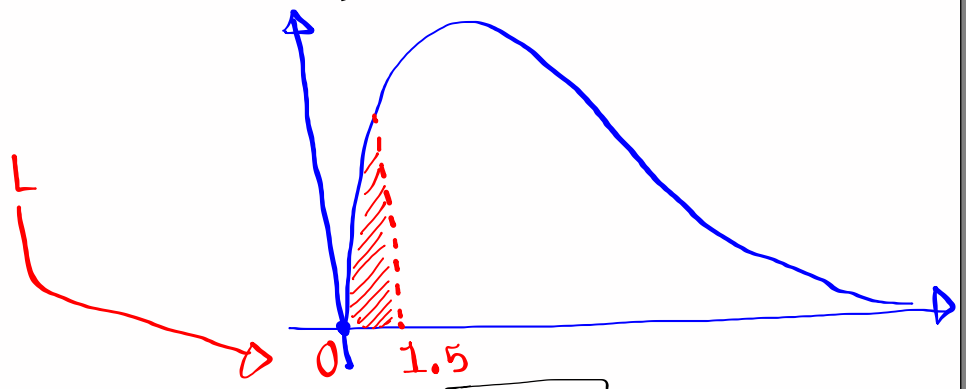
Find $P(F > 10.5)$ with $Ndf = 4$ & $Ddf = 30$.



$fcdf(10.5, E99, 4, 30) = 1.931 \times 10^{-5}$ S.N.

Apr 30-8:56 PM

Find $P(F < 1.5)$ with $Ndf=5$, $Ddf=15$.



$fcdf(0, 1.5, 5, 15) =$ **.752**

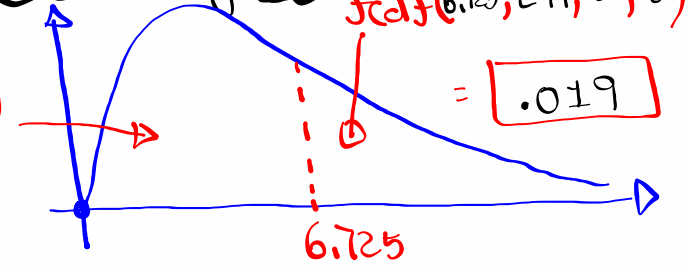
Apr 30-9:03 PM

Given $F = 6.725$, $Ndf = 2$, $Ddf = 8$

Find the area on both sides of F, then multiply smaller area by 2:

$fcdf(0, 6.725, 2, 8) =$ **.981**

$fcdf(6.725, E99, 2, 8) =$ **.019**



$2(.019) =$ **.038**

Apr 30-9:06 PM